

# **3-jet events in DIS at small $x$**

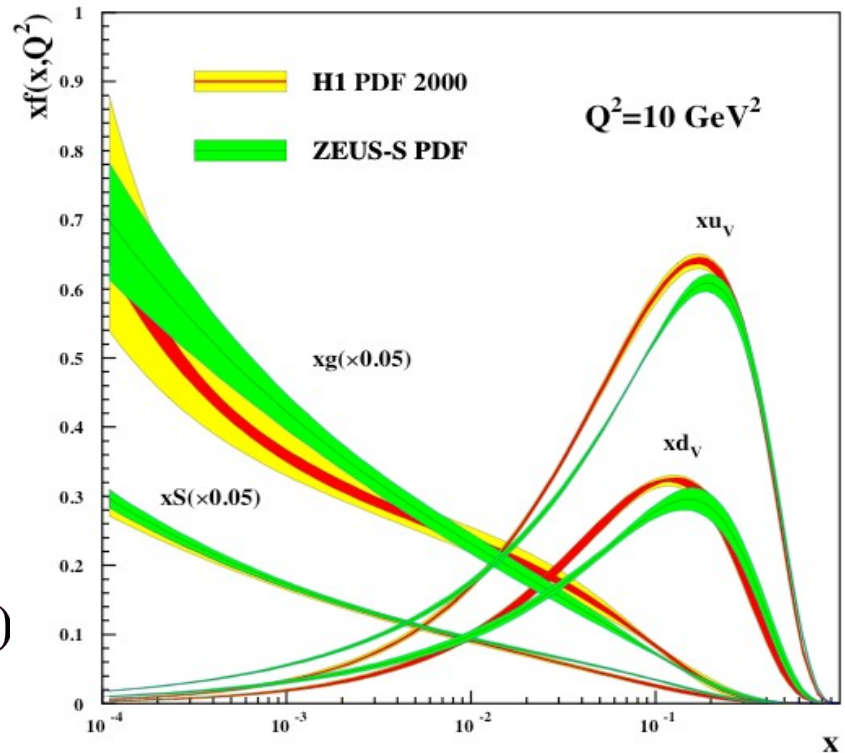
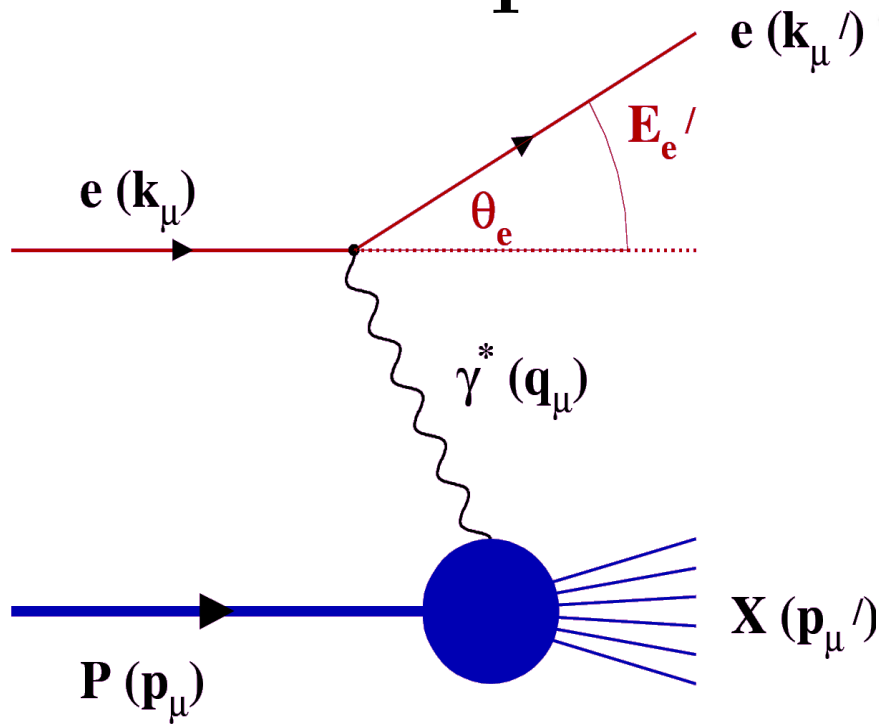
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New York, NY***

***11<sup>th</sup> International Workshop on High- $p_T$  Physics in the RHIC&LHC Era  
12 - 15 April, 2016  
Brookhaven National Laboratory***

**In collaboration with A. Ayala, *M. Hentschinski* and M.E. Tejeda-Yeomans**

# DIS at HERA: parton distributions

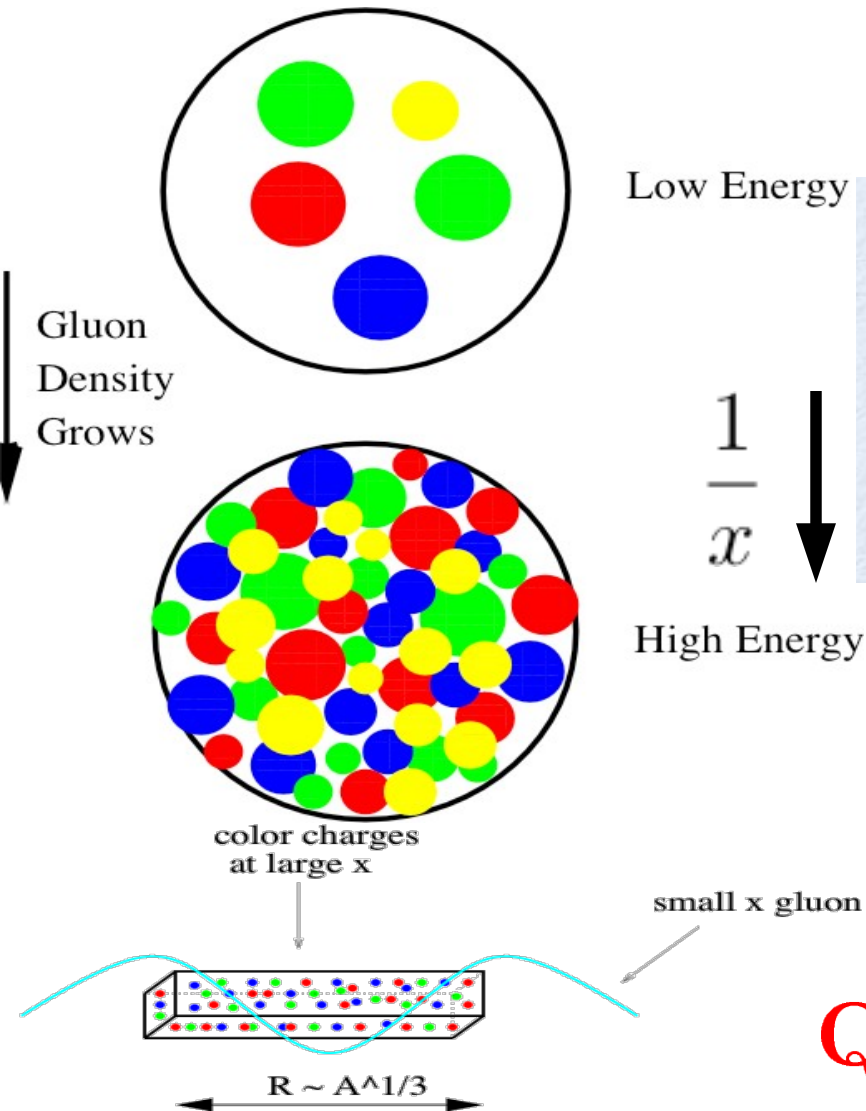
$$Q^2 \equiv -q^2 \quad x = \frac{p^+}{P^+}$$



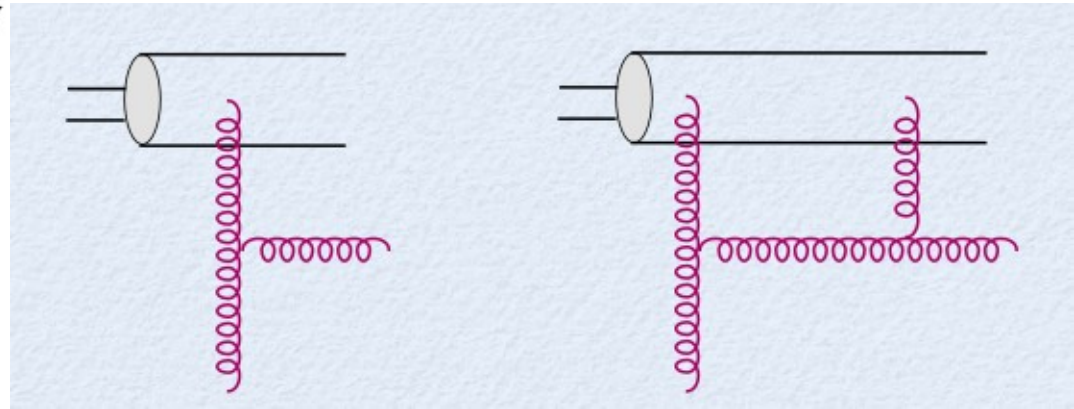
power-like growth of gluon and sea quark distributions with  $x$   
**new QCD dynamics at small  $x$ ?**

# Gluon saturation

*Gribov-Levin-Ryskin  
Mueller-Qiu*



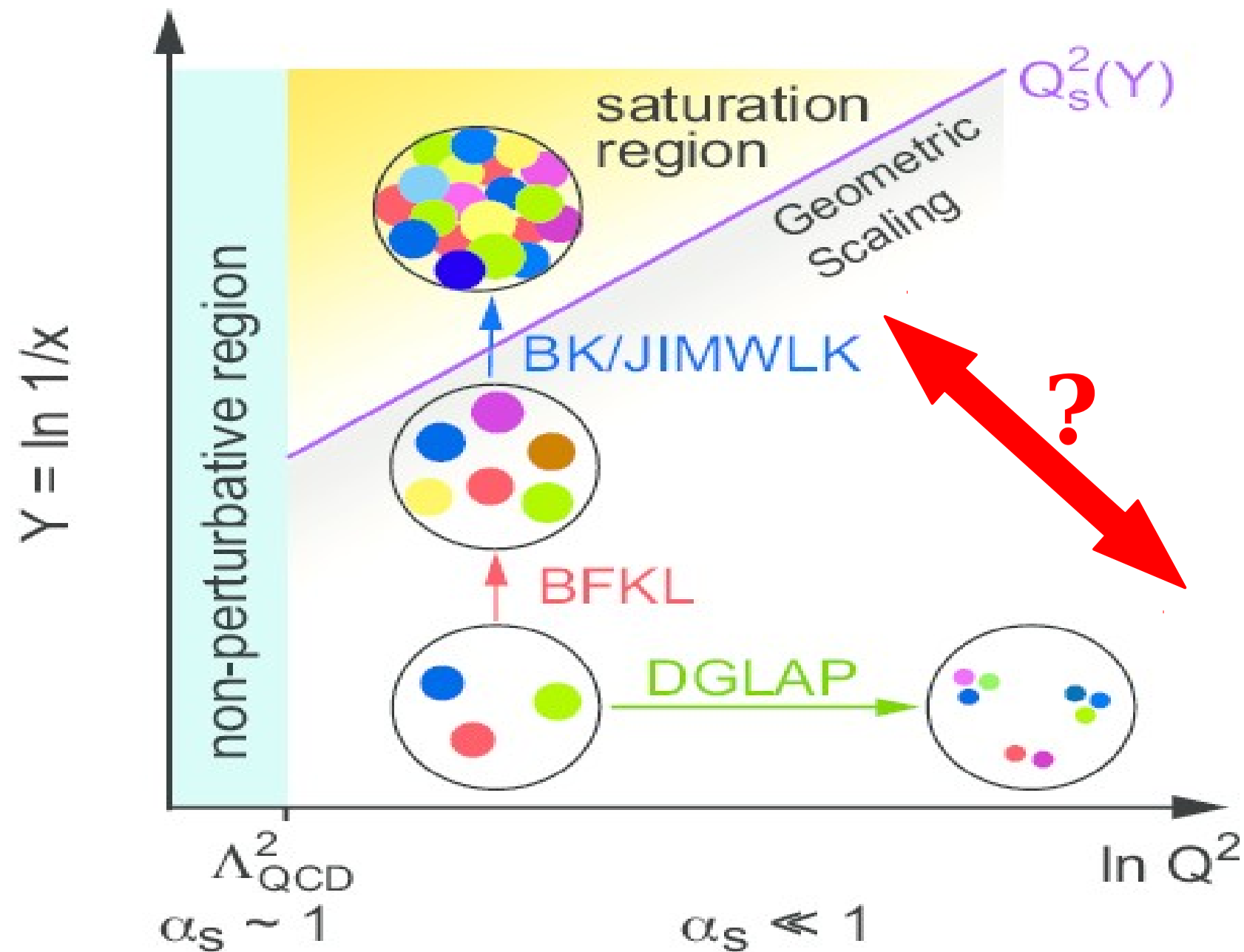
“attractive” bremsstrahlung vs.  
“repulsive” recombination



$$\frac{\alpha_s}{Q^2} \frac{xG(x, b_t, Q^2)}{S_\perp} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

# A proton at high energy: *saturation*



# ***MV effective Action + RGE***

$$S[A, \rho] = -\frac{1}{4} \int d^4x F_{\mu\nu}^2 + \frac{i}{N_c} \int d^2x_t dx^- \delta(x^-) \text{Tr}[\rho(x_t) U(A^-)]$$

**Large x: color source  $\rho$**  **small x: gluon field  $A^\mu$**

$$U(A^-) = \hat{P} \text{Exp} \left[ ig \int dx^+ A_a^- T_a \right]$$

$$Z[j] = \int [D\rho] W_{\Lambda^+}[\rho] \left[ \frac{\int^{\Lambda^+} [D A] \delta(A^+) e^{iS[A, \rho] - \int j \cdot A}}{\int^{\Lambda^+} [D A] \delta(A^+) e^{iS[A, \rho]}} \right]$$

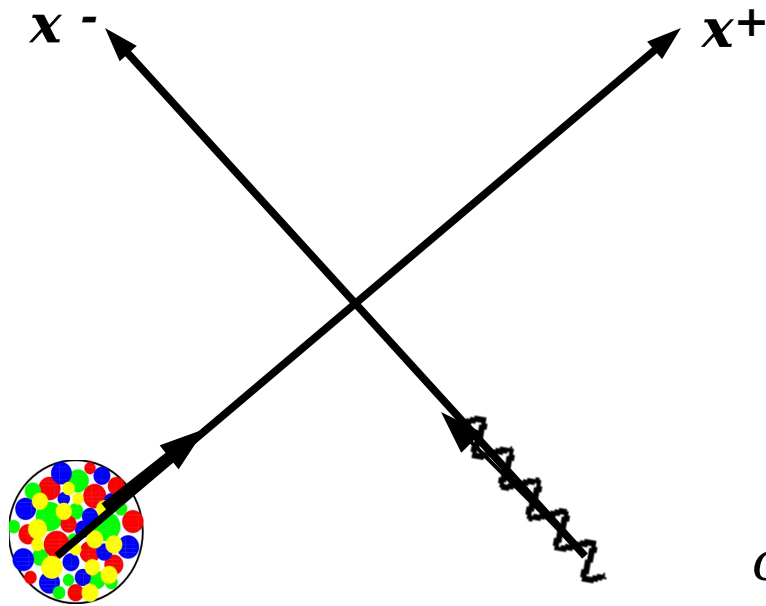
*weight functional:*

**$W_{\Lambda^+}[\rho]$**

*probability distribution of color source  $\rho$   
at longitudinal scale  $\Lambda^+$*

invariance under change of  **$\Lambda^+$**   $\longrightarrow$  RGE for  **$W_{\Lambda^+}[\rho]$**

**Large A/high energy  $\longrightarrow$  saturation**



$$x^+ \equiv \frac{t+z}{\sqrt{2}} \quad x^- \equiv \frac{t-z}{\sqrt{2}}$$

$$\mathbf{J}_a^\mu(\mathbf{x}) \equiv \delta^{\mu+} \delta(\mathbf{x}^-) \rho_a(\mathbf{x}_t)$$

*color current*
*color charge*

*sheet of color charge moving along  $x^+$  and sitting at  $x^- = 0$*

*solution of classical equations of motion*

$$\mathbf{A}_a^+(\mathbf{x}^-, \mathbf{x}_t) = \delta(\mathbf{x}^-) \alpha_a(\mathbf{x}_t)$$

*with*

$$\partial_t^2 \alpha_a(z_t) = g \rho_a(z_t)$$

# ***low $x$ QCD in a background field: CGC***

*(a high gluon density environment)*

## ***two main effects:***

*“multiple scatterings” encoded in classical field ( **$p_t$  broadening**)*

*evolution with  $\ln(1/x)$  a la BK/JIMWLK equation (**suppression**)*

## ***LT pQCD with collinear factorization:***

*single scattering*

*evolution with  $\ln Q^2$*

# ***Signatures***

***dense-dense (AA, pA, pp) collisions***

*multiplicities, spectra*

*long range rapidity correlations*

***dilute-dense (pA, forward pp ) collisions***

*multiplicities*

*$p_t$  spectra*

*angular correlations*

***DIS***

*structure functions (diffraction)*

***NLO di-hadron correlations***

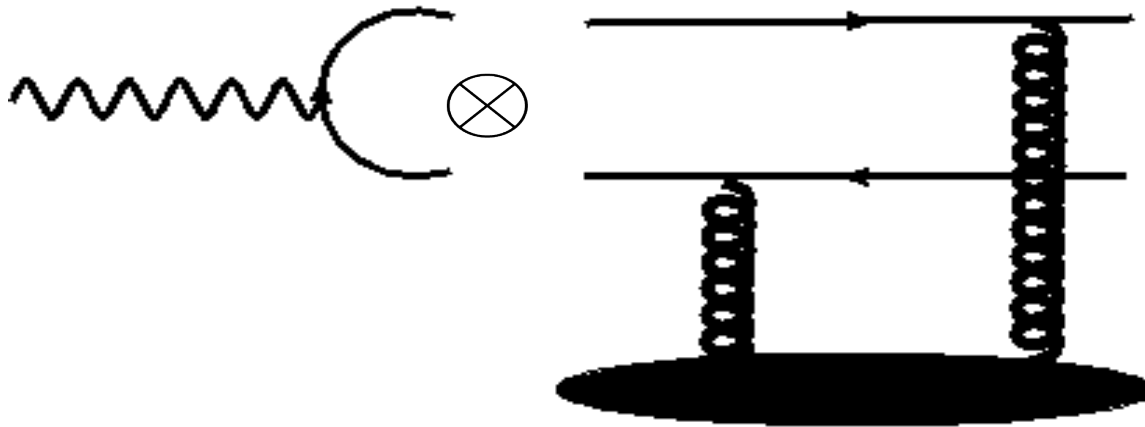
***3-hadron correlations***

***spin asymmetries***

# DIS total cross section

$$\sigma_{\text{DIS}}^{\text{total}} = 2 \int_0^1 dz \int d^2x_t d^2y_t |\Psi(k^\pm, k_t | z, x_t, y_t)|^2 T(x_t, y_t)$$

**dipole cross section**  $T(x_t, y_t) \equiv \frac{1}{N_c} \text{Tr} \langle 1 - V(x_t) V^\dagger(y_t) \rangle$



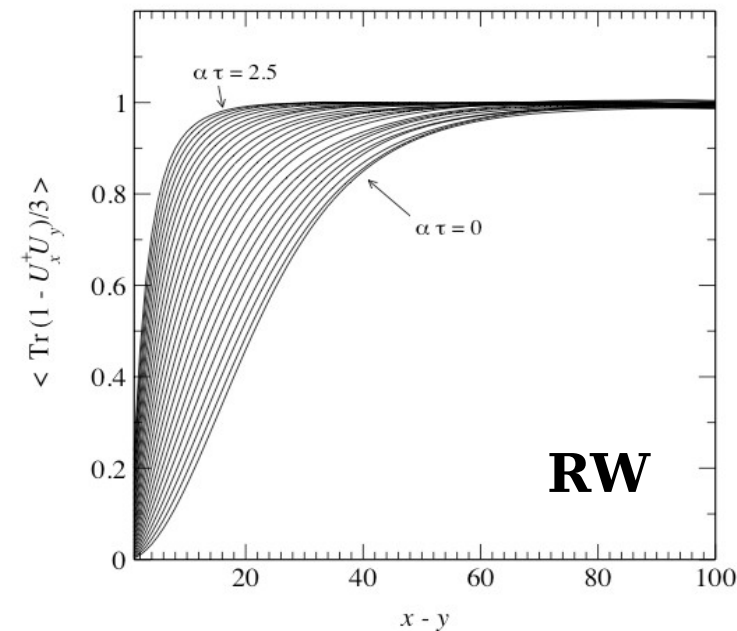
$$V(x_t) \equiv \text{Wilson line} \equiv \text{series of loops} \sim 1 + O(g A) + O(g^2 A^2)$$

*Wilson line encodes multiple scatterings from the color field of the target*

# Dipoles at large $N_c$ : BK eq.

$$\frac{d}{dy} T(\mathbf{x}_t - \mathbf{y}_t) = \frac{\bar{\alpha}_s}{2\pi} \int d^2 \mathbf{z}_t \frac{(\mathbf{x}_t - \mathbf{y}_t)^2}{(\mathbf{x}_t - \mathbf{z}_t)^2 (\mathbf{y}_t - \mathbf{z}_t)^2} \times$$

$$[T(\mathbf{x}_t - \mathbf{z}_t) + T(\mathbf{z}_t - \mathbf{y}_t) - T(\mathbf{x}_t - \mathbf{y}_t) - \mathbf{T}(\mathbf{x}_t - \mathbf{z}_t)\mathbf{T}(\mathbf{z}_t - \mathbf{y}_t)]$$



$$\tilde{T}(\mathbf{p}_t) \rightarrow \log \left[ \frac{Q_s^2}{p_t^2} \right] \quad \text{saturation region}$$

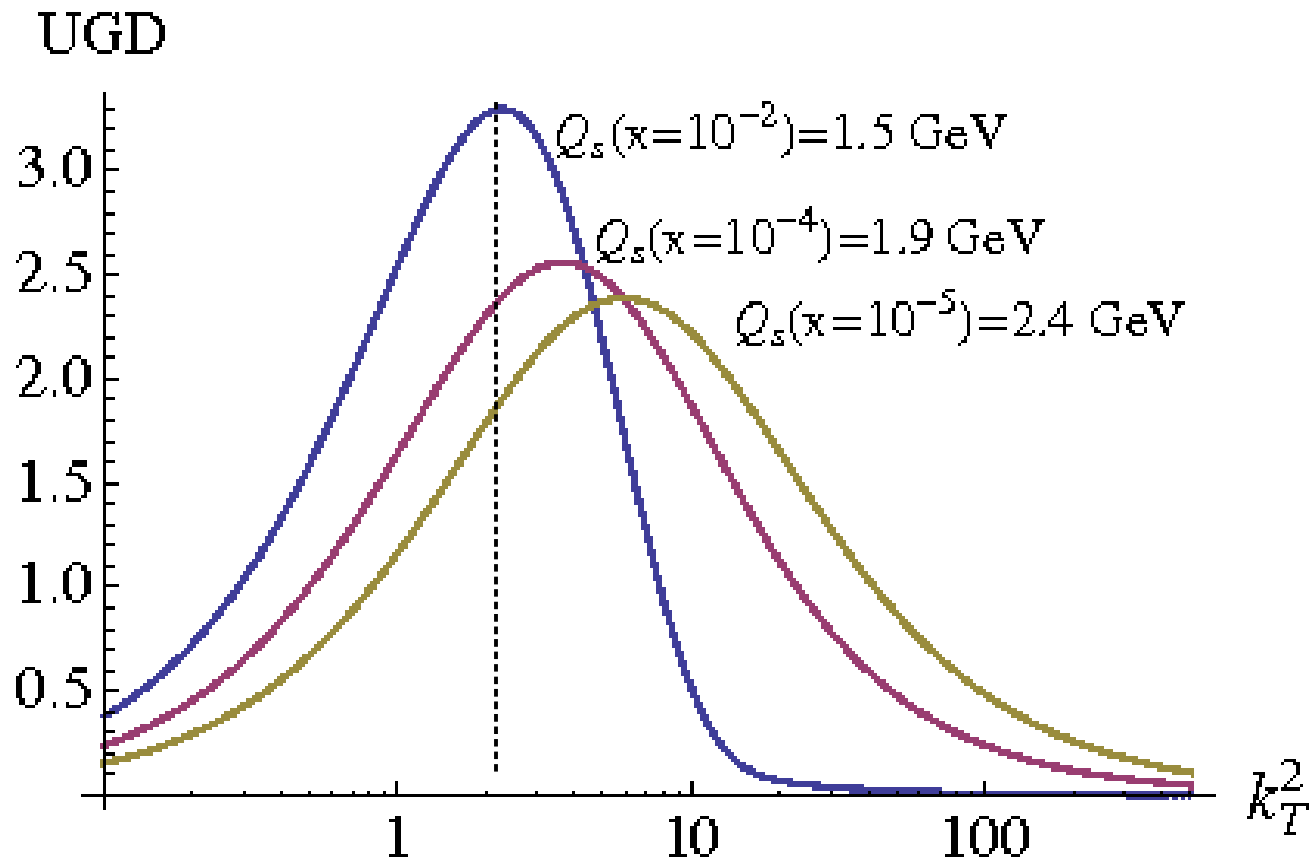
$$\tilde{T}(\mathbf{p}_t) \rightarrow \frac{1}{p_t^2} \left[ \frac{Q_s^2}{p_t^2} \right]^\gamma \quad \text{extended scaling region}$$

$$\tilde{T}(\mathbf{p}_t) \rightarrow \frac{1}{p_t^2} \left[ \frac{Q_s^2}{p_t^2} \right] \quad \text{pQCD region}$$

**Rummukainen-Weigert**, NPA739 (2004) 183

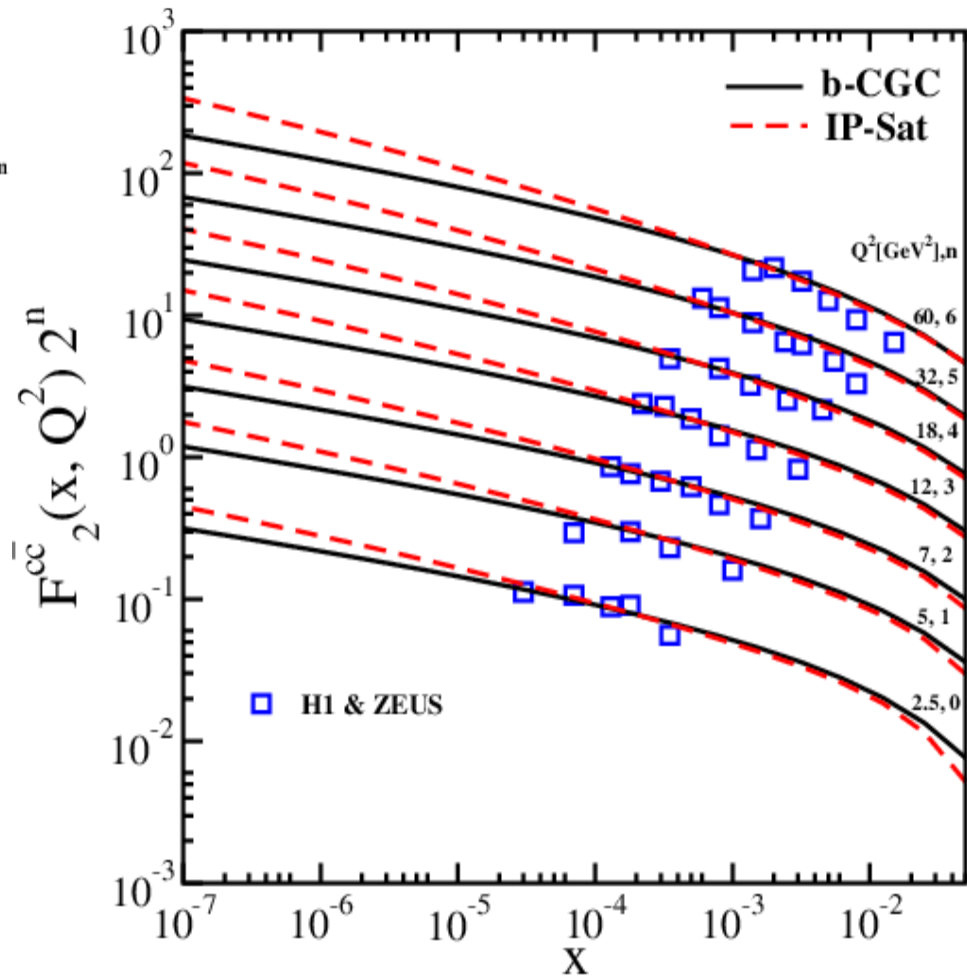
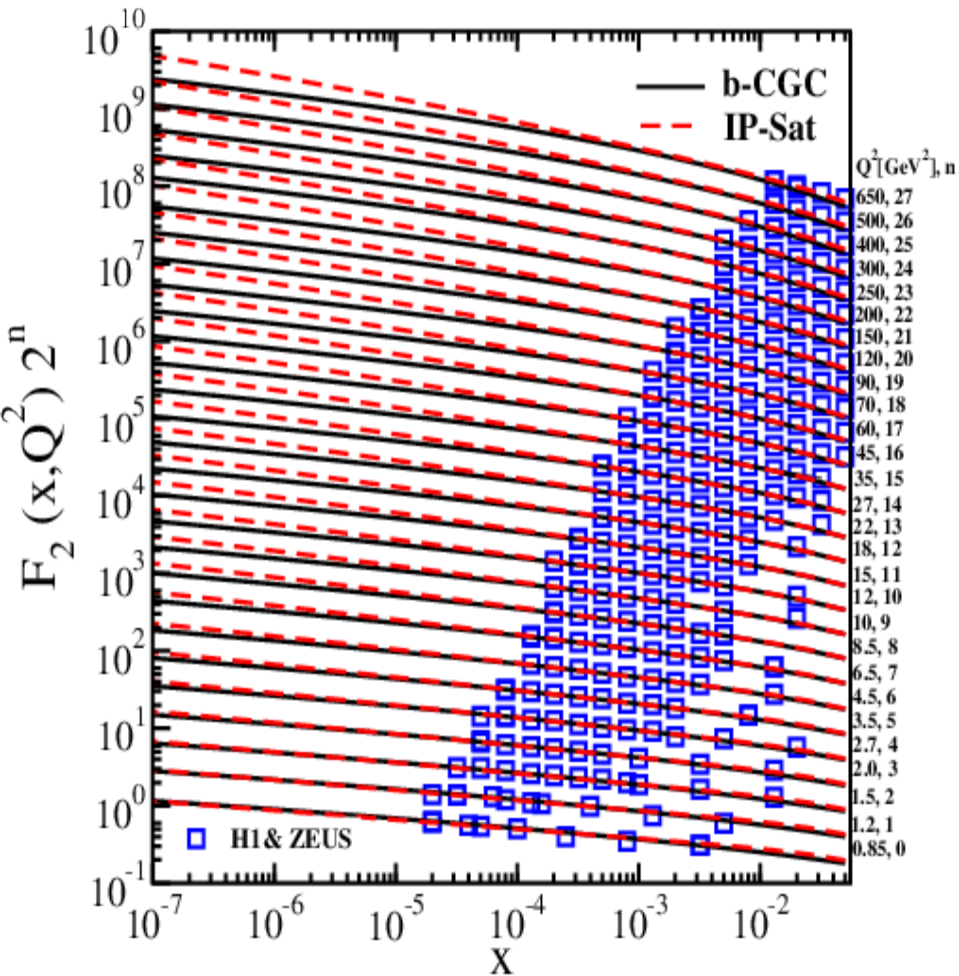
**NLO**: Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)

**unintegrated gluon distribution (ugd):**  $k_t^2 \tilde{T}(k_t)$



multiple scattering: broadening of the peak  
 x-evolution: reduction of magnitude

# HERA ( $ep \rightarrow e X$ )

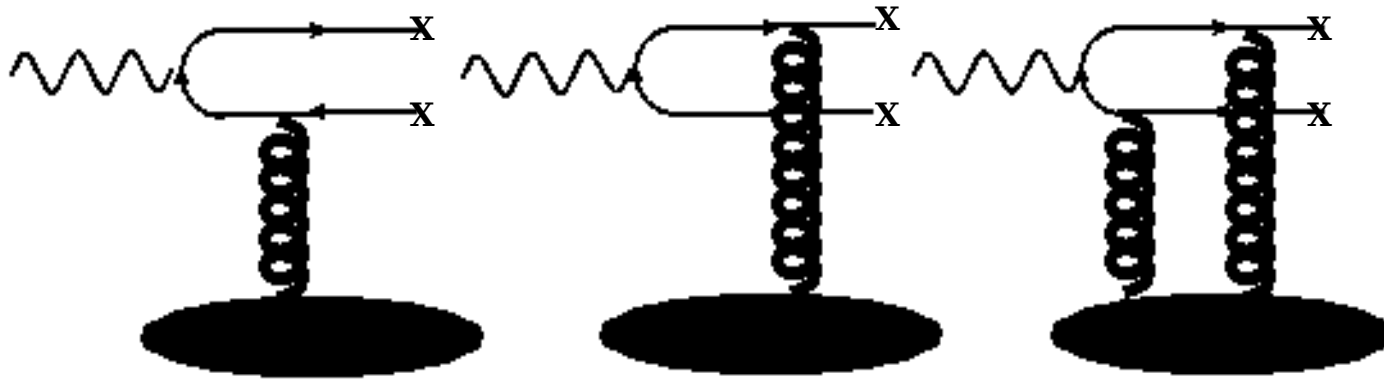


A. Rezaeian and I. Schmidt, PRD88 (2013) 074016

$Q_s^2 \sim 1 \text{ GeV}^2$  for  $x \sim 3 \times 10^{-4}$

***something with more discriminating power***  
*di-hadron (azimuthal) angular correlations in DIS*

**LO:**  $\gamma^*(\mathbf{k}) \text{ p} \rightarrow \text{q}(\mathbf{p}) \bar{\text{q}}(\mathbf{q}) \text{ X}$



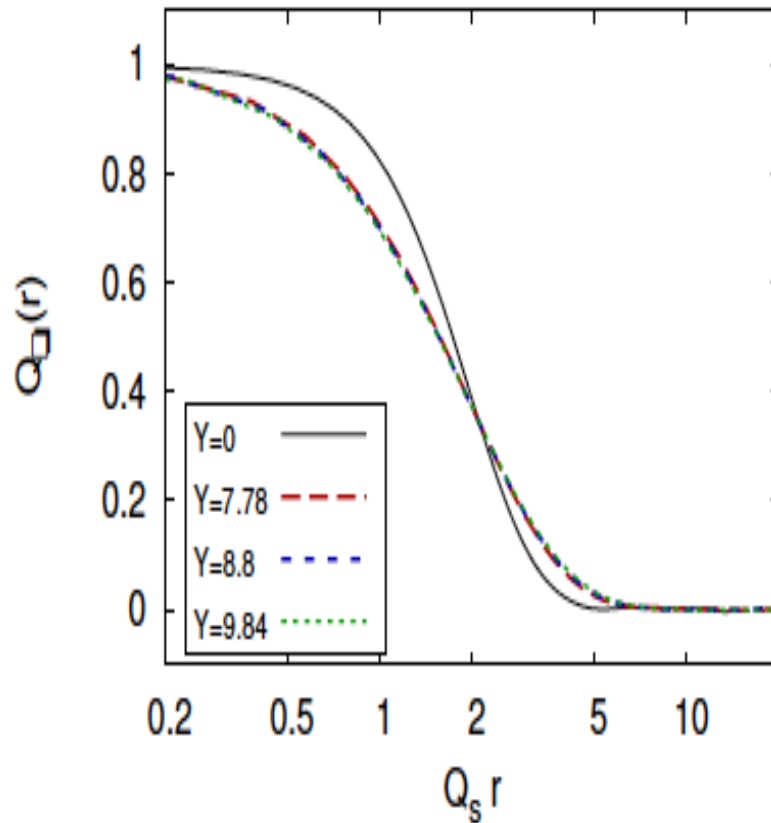
**new d.o.f**  
**quadrupoles**

quark propagator in the background color field

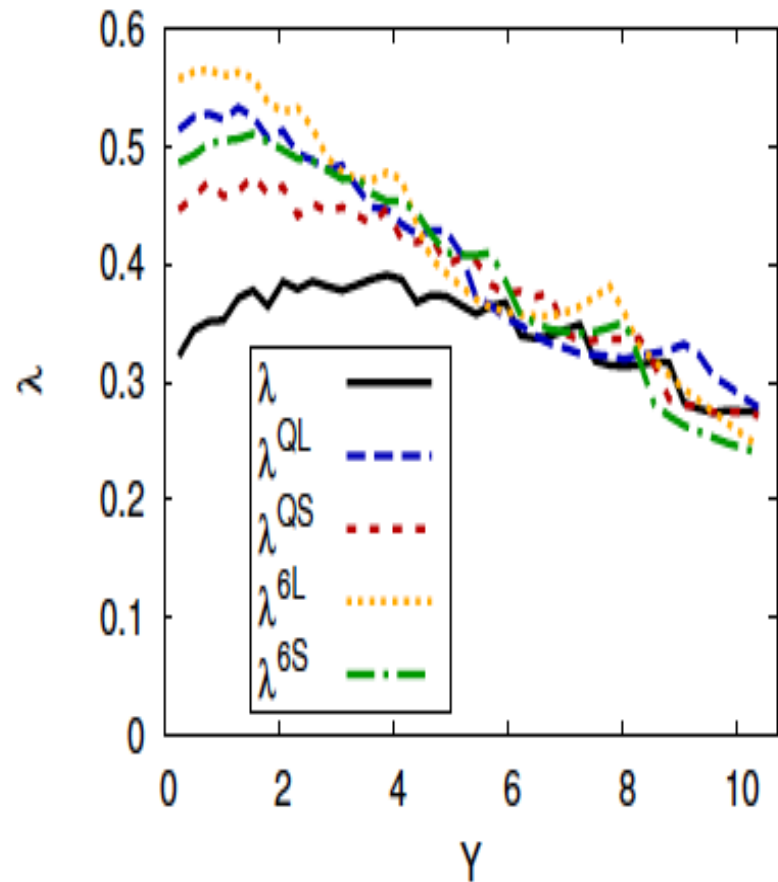
$$S_F(q, p) \equiv (2\pi)^4 \delta^4(p - q) S_F^0(p) + S_F^0(q) \tau_f(q, p) S_F^0(p)$$

$$\begin{aligned} \tau_f(q, p) &\equiv (2\pi)\delta(p^- - q^-) \gamma^- \int d^2x_t e^{i(q_t - p_t) \cdot x_t} \\ &\quad \{ \theta(p^-)[V(x_t) - 1] - \theta(-p^-)[V^\dagger(x_t) - 1] \} \end{aligned}$$

# Quadrupole evolution: JIMWLK

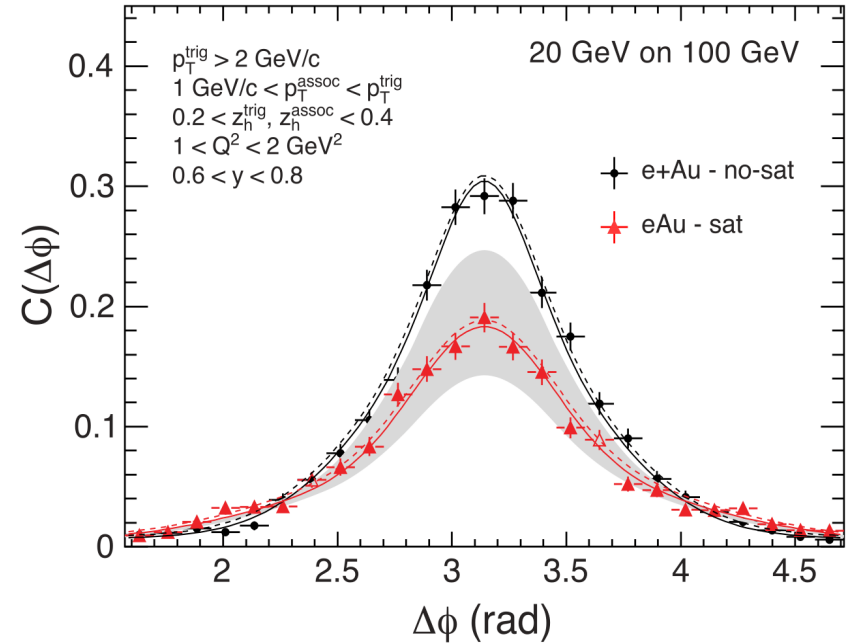
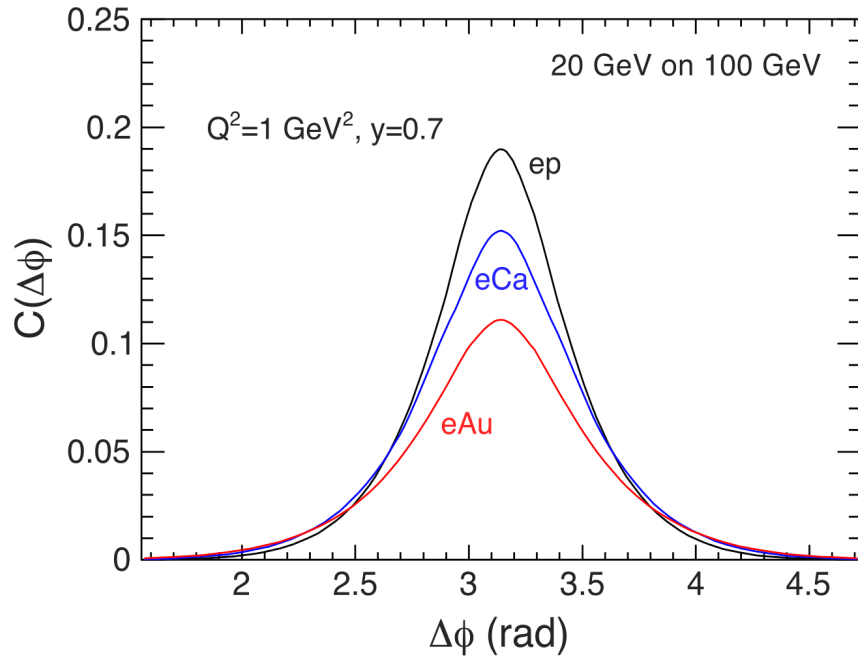


*Geometric scaling also present in quadrupoles*



*Energy dependence of saturation scale*

# Di-hadron azimuthal correlations in DIS



*Electron Ion Collider...., A. Accardi et al., arXiv:1212.1701*

*Zheng-Aschenauer-Lee-Xiao, PRD89 (2014)7, 074037*

# Precision CGC: NLO *corrections*

DIS total cross section:

photon impact factor  
evolution equations

pA collisions:

Single inclusive particle production

**NLO di-jet production in DIS**

***LO 3-jet production***

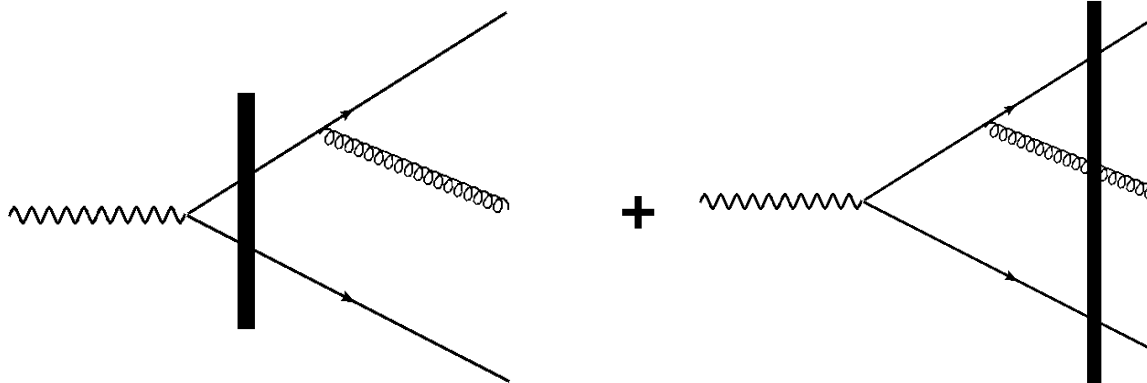
*two away side hadrons: additional knob*

# Azimuthal correlations in DIS

## *di-jet production in DIS: **NLO***

*real contributions:*  $\gamma^* \mathbf{T} \rightarrow q \bar{q} g \mathbf{X}$

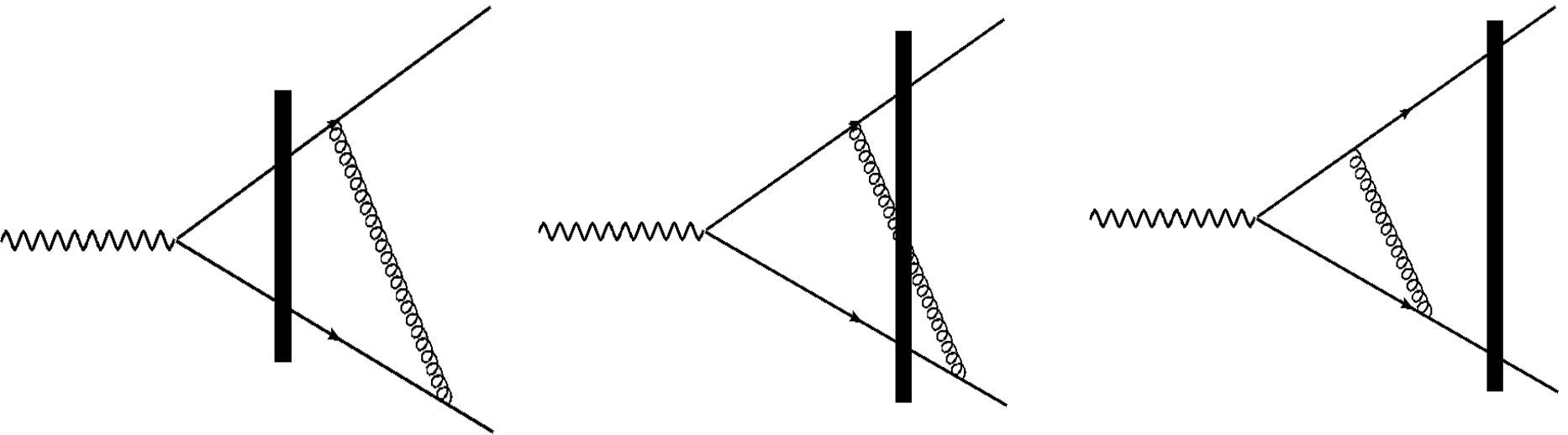
*integrate out one of the produced partons*



*work in progress: Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans*

# di-jet azimuthal correlations in DIS

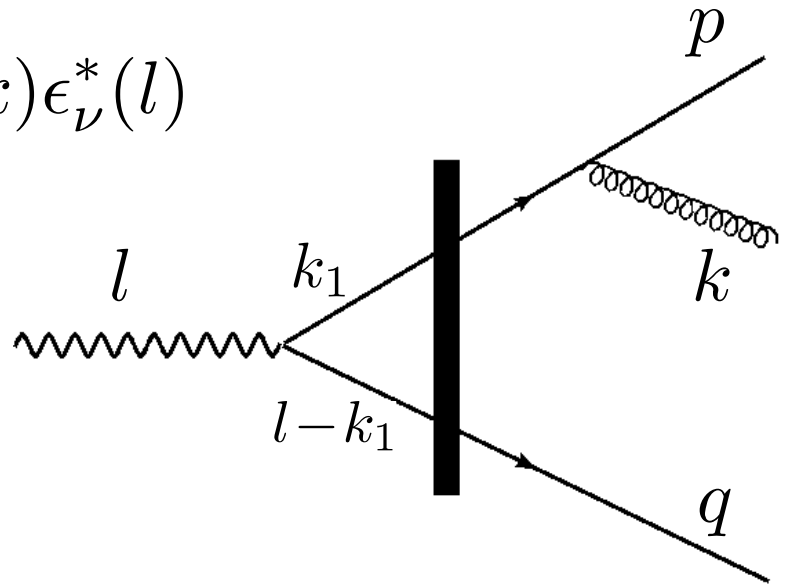
*virtual contributions:*  $\gamma^* \mathbf{T} \rightarrow \mathbf{q} \bar{\mathbf{q}} \mathbf{X}$



+ “*self-energy*” diagrams

**real contributions:**

$$\mathcal{A} \equiv -eg \bar{u}(p) [A]^{\mu\nu} v(q) \epsilon_\mu(k) \epsilon_\nu^*(l)$$



$$A_1^{\mu\nu} = \gamma^\mu t^a S_F^0(p+k) \tau_F(p+k, k_1) S_F^0(k_1) \gamma^\nu S_F^0(l-k_1) \tau_F(l-k_1, q) \frac{d^4 k_1}{(2\pi)^4}$$

$$= \frac{i}{2l^-} \frac{\delta(l^- - p^- - q^- - k^-)}{(p+k)^2} \int d^2 x_t d^2 y_t e^{-i(p_t+k_t) \cdot x_t} e^{-iq_t \cdot y_t}$$

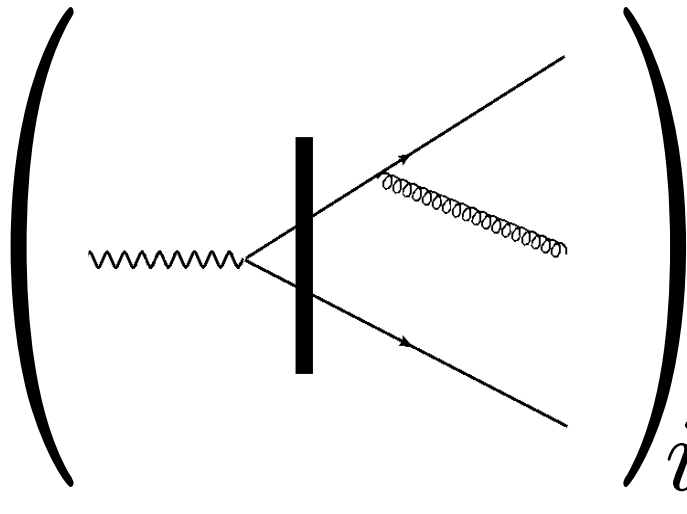
$$\gamma^\mu t^a i(\not{p} + \not{k}) \gamma^- i\not{k}_1 \gamma^\nu i(\not{l} - \not{k}_1) \gamma^- K_0 [L(x_t - y_t)]$$

$$V(x_t) V^\dagger(y_t)$$

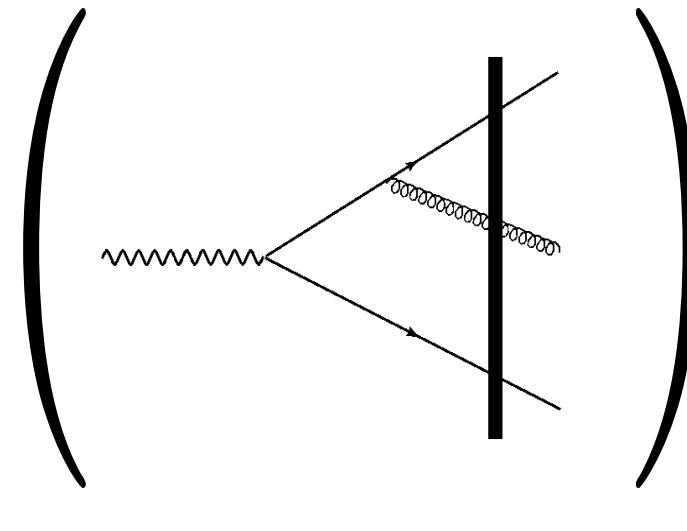
with

$$L^2 = \frac{q^-(p^- + k^-)}{l^- l^-} Q^2 \quad k_1^- = p^- - k^- \quad k_1^+ = \frac{k_{1t}^2 - i\epsilon}{2(p^- + k^-)} \quad k_{1t} = -i \partial_{x_t - y_t}$$

# structure of Wilson lines: amplitude



$$\left( \text{diagram} \right)_{ij} = [V^\dagger(y_t) V(x_t) t^a]_{ij}$$



$$\left( \text{diagram} \right)_{ij} = [V^\dagger(y_t) V(x_t) t^b]_{ij} U^{ba}(z_t)$$

# structure of Wilson lines: cross section

$$\begin{aligned}
\text{tr}[W_1 W_1^*] &= \frac{(N_c^2 - 1) S_Q(x_t, x'_t, y'_t, y_t)}{2N_c} \\
\text{tr}[W_1 W_2^*] &= \frac{1}{4} \left( S_D(z'_t, x'_t) S_Q(x_t, z'_t, y'_t, y_t) - \frac{S_Q(x_t, x'_t, y'_t, y_t)}{N_c} \right) \\
\text{tr}[W_1 W_3^*] &= \frac{1}{2} \left( S_D(x_t, y) S_D(y'_t, x'_t) - \frac{S_Q(x_t, x'_t, y'_t, y_t)}{N_c} \right) \\
\text{tr}[W_1 W_4^*] &= \frac{1}{4} \left( S_D(z'_t, x'_t) S_Q(x_t, z'_t, y'_t, y_t) - \frac{S_Q(x_t, x'_t, y'_t, y_t)}{N_c} \right) \\
\text{tr}[W_2 W_1^*] &= \frac{1}{4} \left( S_D(x_t, z) S_Q(z_t, x'_t, y'_t, y_t) - \frac{S_Q(x_t, x'_t, y'_t, y_t)}{N_c} \right) \\
\text{tr}[W_2 W_2^*] &= \frac{1}{8} \left( S_Q(x_t, x'_t, z'_t, z_t) S_Q(z, z'_t, y'_t, y_t) - \frac{S_Q(x_t, x'_t, y'_t, y_t)}{N_c} \right) \\
\text{tr}[W_2 W_3^*] &= \frac{1}{4} \left( S_D(z, y_t) S_Q(x_t, x'_t, y'_t, z) - \frac{S_Q(x_t, x'_t, y'_t, y_t)}{N_c} \right) \\
\text{tr}[W_2 W_4^*] &= \frac{1}{8} \left( S_Q(x_t, x'_t, z'_t, z) S_Q(z_t, z'_t, y'_t, y_t) - \frac{S_Q(x_t, x'_t, y'_t, y_t)}{N_c} \right) \\
\text{tr}[W_3 W_1^*] &= \frac{1}{2} \left( S_D(x_t, y_t) S_D(y'_t, x'_t) - \frac{S_Q(x_t, x'_t, y'_t, y_t)}{N_c} \right) \\
\text{tr}[W_3 W_2^*] &= \frac{1}{4} \left( S_D(y'_t, z'_t) S_Q(x_t, x'_t, z'_t, y_t) - \frac{S_Q(x_t, x'_t, y'_t, y_t)}{N_c} \right) \\
\text{tr}[W_3 W_3^*] &= \frac{(N_c^2 - 1) S_Q(x_t, x'_t, y'_t, y_t)}{2N_c} \\
\text{tr}[W_3 W_4^*] &= \frac{1}{4} \left( S_D(y'_t, z'_t) S_Q(x_t, x'_t, z'_t, y_t) - \frac{S_Q(x_t, x'_t, y'_t, y_t)}{N_c} \right) \\
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\end{aligned}$$

$$\gamma^* \mathbf{p} \rightarrow \mathbf{q} \bar{\mathbf{q}} \mathbf{g} \mathbf{X}$$

we are  
developing a  
Mathematica  
package  
to put all this  
together

Traces:  
brute force ~ 23 pages long!

**Something more clever:  
spinor helicity methods**

$A1^2 =$

$$+ q_{\text{minus}} * ( \text{DENn}(k) * \text{dot}(p, k) * \text{IntR1}(n_{\text{minus}}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) * \\ \text{IntR1c}(\text{muc1}, \text{muc1}, n_{\text{minus}}, 1, 1, 1, p) + \text{DENn}(k) * \text{dot}(p, k) * \text{IntR1}(n_{\text{minus}}, \\ n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}(\text{muc2}, \text{muc2}, n_{\text{minus}}, 1, 1, 1, p) - \text{DENn}(k) * \\ \text{dot}(p, k) * \text{IntR1}(n_{\text{minus}}, \text{mu2}, n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}(n_{\text{minus}}, \text{mu2}, n_{\text{minus}}, 1, \\ 1, 1, p) - \text{DENn}(k) * \text{dot}(p, k) * \text{IntR1}(n_{\text{minus}}, \text{muc2}, n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}( \\ n_{\text{minus}}, \text{muc2}, n_{\text{minus}}, 1, 1, 1, p) - \text{DENn}(k) * \text{dot}(p, k) * \text{IntR1}(\text{mu1}, n_{\text{minus}}, \\ n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}(\text{mu1}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) + \text{DENn}(k) * \text{dot}(p, k) * \\ \text{IntR1}(\text{mu1}, \text{mu1}, n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}(n_{\text{minus}}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) + \\ \text{DENn}(k) * \text{dot}(p, k) * \text{IntR1}(\text{mu2}, \text{mu2}, n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}(n_{\text{minus}}, n_{\text{minus}}, \\ n_{\text{minus}}, 1, 1, 1, p) - \text{DENn}(k) * \text{dot}(p, k) * \text{IntR1}(\text{muc1}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) * \\ \text{IntR1c}(\text{muc1}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) - \text{IntR1}(n_{\text{minus}}, n_{\text{minus}}, p, 1, 1, 1, p) * \\ \text{IntR1c}(\text{muc1}, \text{muc1}, n_{\text{minus}}, 1, 1, 1, p) + \text{IntR1}(n_{\text{minus}}, n_{\text{minus}}, p, 1, 1, 1, p) * \\ \text{IntR1c}(\text{muc2}, \text{muc2}, n_{\text{minus}}, 1, 1, 1, p) + \text{IntR1}(n_{\text{minus}}, \text{mu2}, p, 1, 1, 1, p) * \\ \text{IntR1c}(n_{\text{minus}}, \text{mu2}, n_{\text{minus}}, 1, 1, 1, p) - \text{IntR1}(n_{\text{minus}}, \text{muc2}, p, 1, 1, 1, p) * \\ \text{IntR1c}(n_{\text{minus}}, \text{muc2}, n_{\text{minus}}, 1, 1, 1, p) + \text{IntR1}(\text{mu1}, p, \text{mu1}, 1, 1, 1, p) * \text{IntR1c}( \\ n_{\text{minus}}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) - \text{IntR1}(\text{mu1}, n_{\text{minus}}, p, 1, 1, 1, p) * \text{IntR1c}( \\ \text{mu1}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) - \text{IntR1}(\text{mu1}, n_{\text{minus}}, \text{mu1}, 1, 1, 1, p) * \text{IntR1c}(p, \\ n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) + \text{IntR1}(\text{mu1}, \text{mu1}, p, 1, 1, 1, p) * \text{IntR1c}(n_{\text{minus}}, \\ n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) - \text{IntR1}(\text{mu2}, \text{mu2}, p, 1, 1, 1, p) * \text{IntR1c}(n_{\text{minus}}, \\ n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) - \text{IntR1}(\text{mu3}, p, \text{mu3}, 1, 1, 1, p) * \text{IntR1c}(n_{\text{minus}}, \\ n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) + \text{IntR1}(\text{mu3}, n_{\text{minus}}, \text{mu3}, 1, 1, 1, p) * \text{IntR1c}(p, \\ n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) + \text{IntR1}(\text{muc1}, n_{\text{minus}}, p, 1, 1, 1, p) * \text{IntR1c}(\text{muc1}, \\ n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) )$$

$$+ p_{\text{minus}} * q_{\text{minus}} * ( - \text{DENn}(k) * \text{IntR1}(k, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}( \\ \text{muc3}, n_{\text{minus}}, \text{muc3}, 1, 1, 1, p) + \text{DENn}(k) * \text{IntR1}(k, n_{\text{minus}}, \text{mu3}, 1, 1, 1, p) * \\ \text{IntR1c}(\text{mu3}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) - \text{DENn}(k) * \text{IntR1}(k, \text{mu3}, \text{mu3}, 1, 1, 1, p) * \\ \text{IntR1c}(n_{\text{minus}}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) + \text{DENn}(k) * \text{IntR1}(k, \text{muc3}, n_{\text{minus}}, 1, \\ 1, 1, p) * \text{IntR1c}(n_{\text{minus}}, n_{\text{minus}}, \text{muc3}, 1, 1, 1, p) + \text{DENn}(k) * \text{IntR1}(n_{\text{minus}}, k, \\ n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}(n_{\text{minus}}, \text{muc3}, \text{muc3}, 1, 1, 1, p) - \text{DENn}(k) * \text{IntR1}( \\ n_{\text{minus}}, k, \text{mu3}, 1, 1, 1, p) * \text{IntR1c}(n_{\text{minus}}, \text{mu3}, n_{\text{minus}}, 1, 1, 1, p) + \text{DENn}(k) * \\ \text{IntR1}(n_{\text{minus}}, n_{\text{minus}}, k, 1, 1, 1, p) * \text{IntR1c}(\text{muc1}, \text{muc1}, n_{\text{minus}}, 1, 1, 1, p) - \\ \text{DENn}(k) * \text{IntR1}(n_{\text{minus}}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}(k, \text{muc3}, \text{muc3}, 1, 1, 1, \\ p) + \text{DENn}(k) * \text{IntR1}(n_{\text{minus}}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}(\text{muc2}, \text{muc2}, k, 1, \\ 1, 1, p) + \text{DENn}(k) * \text{IntR1}(n_{\text{minus}}, n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}(\text{muc3}, k, \\ \text{muc3}, 1, 1, 1, p) + \text{DENn}(k) * \text{IntR1}(n_{\text{minus}}, n_{\text{minus}}, \text{mu3}, 1, 1, 1, p) * \text{IntR1c}(k, \text{mu3}, \\ n_{\text{minus}}, 1, 1, 1, p) - \text{DENn}(k) * \text{IntR1}(n_{\text{minus}}, n_{\text{minus}}, \text{mu3}, 1, 1, 1, p) * \text{IntR1c}( \\ \text{mu3}, k, n_{\text{minus}}, 1, 1, 1, p) - \text{DENn}(k) * \text{IntR1}(n_{\text{minus}}, \text{mu2}, k, 1, 1, 1, p) * \text{IntR1c}( \\ n_{\text{minus}}, \text{mu2}, n_{\text{minus}}, 1, 1, 1, p) + \text{DENn}(k) * \text{IntR1}(n_{\text{minus}}, \text{mu3}, \text{mu3}, 1, 1, 1, p) * \\ \text{IntR1c}(n_{\text{minus}}, k, n_{\text{minus}}, 1, 1, 1, p) - \text{DENn}(k) * \text{IntR1}(n_{\text{minus}}, \text{muc2}, n_{\text{minus}}, 1, \\ 1, 1, p) * \text{IntR1c}(n_{\text{minus}}, \text{muc2}, k, 1, 1, 1, p) - \text{DENn}(k) * \text{IntR1}(n_{\text{minus}}, \text{muc3}, \\ n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}(n_{\text{minus}}, k, \text{muc3}, 1, 1, 1, p) - \text{DENn}(k) * \text{IntR1}(\text{mu1}, \\ n_{\text{minus}}, n_{\text{minus}}, 1, 1, 1, p) * \text{IntR1c}(\text{mu1}, n_{\text{minus}}, k, 1, 1, 1, p) + \text{DENn}(k) * \text{IntR1}($$

# spinor helicity methods

Review:

L. Dixon, hep-ph/9601359

massless quarks: helicity eigenstates

$$u_{\pm}(k) \equiv \frac{1}{2} (1 \pm \gamma_5) u(k)$$

$$\overline{u_{\pm}(k)} \equiv \overline{u(k)} \frac{1}{2} (1 \mp \gamma_5)$$

$$v_{\mp}(k) \equiv \frac{1}{2} (1 \pm \gamma_5) v(k)$$

$$\overline{v_{\mp}(k)} \equiv \overline{v(k)} \frac{1}{2} (1 \mp \gamma_5)$$

$$u_+(k) = v_-(k) = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \\ \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \end{bmatrix}$$

$$u_-(k) = v_+(k) = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{k^-} e^{-i\phi_k} \\ -\sqrt{k^+} \\ -\sqrt{k^-} e^{-i\phi_k} \\ \sqrt{k^+} \end{bmatrix} \quad \text{with} \quad e^{\pm i\phi_k} \equiv \frac{k_x \pm ik_y}{\sqrt{k^+ k^-}}$$

notation:

$$|i^{\pm} \rangle \equiv |k_i^{\pm} \rangle \equiv u_{\pm}(k_i) = v_{\mp}(k_i)$$

basic spinor products:

$$\langle i j \rangle \equiv \langle i^- | j^+ \rangle = \overline{u_-(k_i)} u_+(k_j)$$

$$[i j] \equiv \langle i^+ | j^- \rangle = \overline{u_+(k_i)} u_-(k_j)$$

$$\langle i^{\pm} | \equiv \langle k_i^{\pm} | \equiv \overline{u_{\pm}(k_i)} = \overline{v_{\mp}(k_i)}$$

with

$$\langle i j \rangle \equiv \sqrt{|s_{ij}|} e^{i\phi_{ij}}$$

where

$$[i j] \equiv \sqrt{|s_{ij}|} e^{-i(\phi_{ij} + \pi)}$$

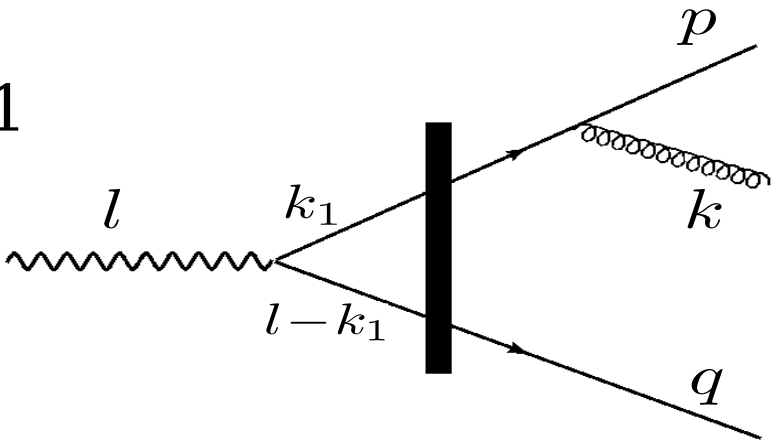
$$\cos \phi_{ij} = \frac{k_i^x k_j^+ - k_j^x k_i^+}{\sqrt{|s_{ij}| k_i^+ k_j^+}}$$

and

$$\sin \phi_{ij} = \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}| k_i^+ k_j^+}}$$

$$s_{ij} = (k_i + k_j)^2 = 2k_i \cdot k_j$$

# Diagram A1



longitudinal photons

gluon, quark helicity

$$g, h = \pm$$

$$A_{1,hg}^L = \sqrt{2Q^2} e^{ix_t(k_t+p_t)+iq_ty_t} K_0 \left[ \sqrt{Q^2 x_{12}^2 z_2 (z_1 + z_3)} \right] \cdot a_{1,hg}^L$$

with  $x_{12}^2 \equiv (x_t - y_t)^2$

$$a_{1,++}^L = \frac{z_1 z_2 \sqrt{z_1 z_2} (z_1 + z_3)}{z_3 e^{-i\theta_p} |p_t| - z_1 e^{-i\theta_k} |k_t|}$$

$$a_{1,-+}^L = \frac{\sqrt{z_1 z_2} z_2 (z_1 + z_3)^2}{z_3 e^{-i\theta_p} |p_t| - z_1 e^{-i\theta_k} |k_t|}$$

$$a_{1,--}^L = (a_{1,++}^L)^*$$

$$a_{1,+-}^L = (a_{1,+--+}^L)^*$$

add up all the pieces, Fourier transform, square the amplitude,....  
triple differential cross section

# ***preliminary: illustration only!***

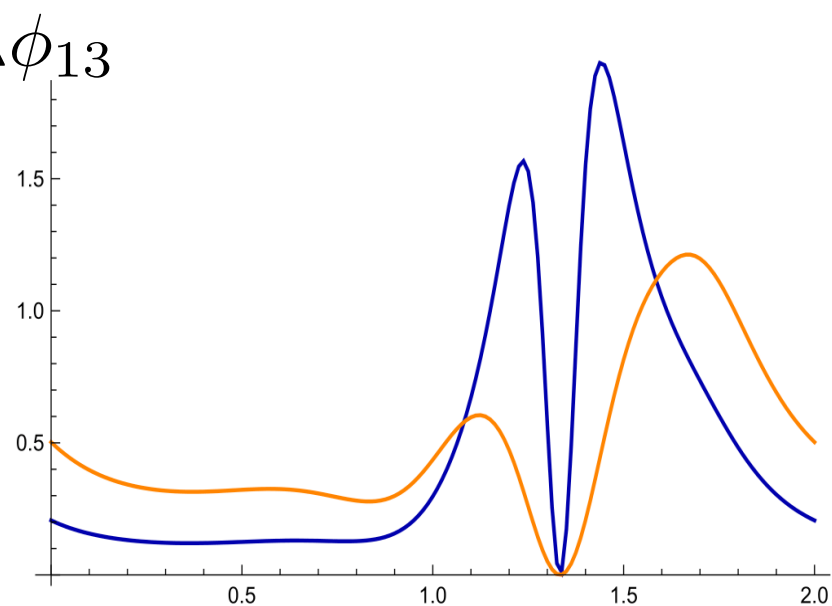
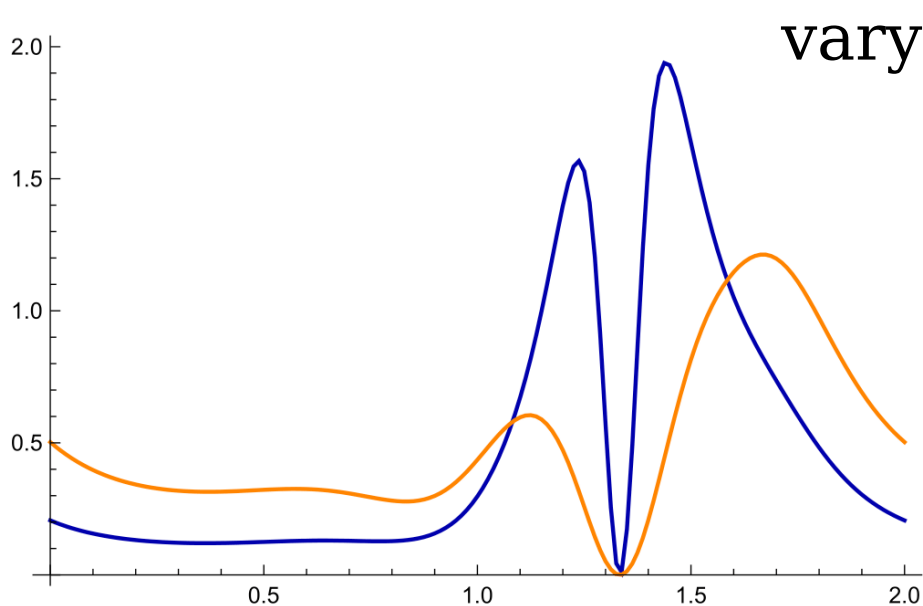
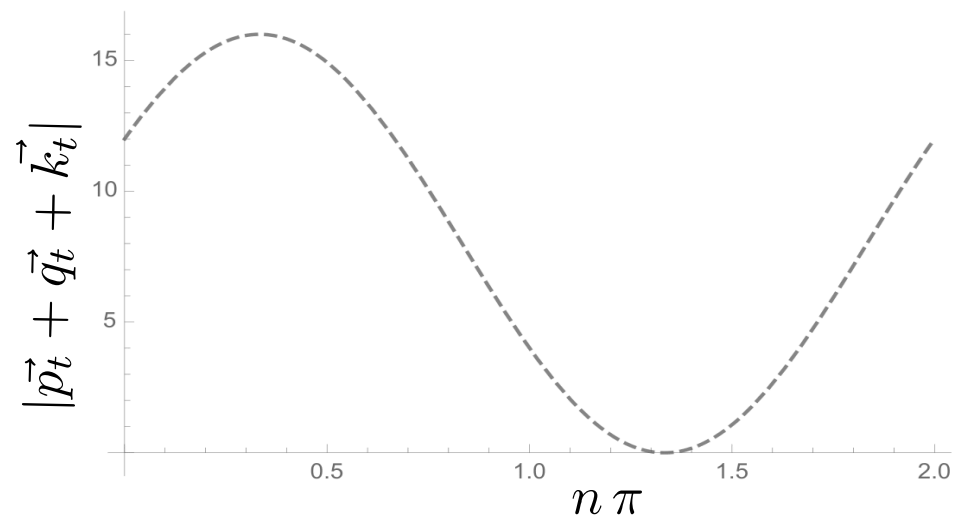
linear regime: use ugd's

$$z_1 = z_2 = 0.2, z_3 = 0.6$$

$$p_t = q_t = k_t = 4 \text{ GeV}$$

$$Q^2 = 16 \text{ GeV}^2$$

$$\Delta\phi_{12} = \frac{2\pi}{3}$$



# di-jet azimuthal correlations in DIS

$$\text{NLO: } \gamma^* \mathbf{p} \rightarrow \mathbf{h h X}$$

*integrate out one of the produced partons - there are divergences:*

*rapidity divergences: JIMWLK evolution of n-point functions*

*collinear divergences: DGLAP evolution of fragmentation functions*

*infrared divergences cancel*

***the finite pieces are written as a factorized cross section***

*related work by:*

*Boussarie, Grabovsky, Szymanowski, Wallon, JHEP1409, 026 (2014)*

*Balitsky, Chirilli, PRD83 (2011) 031502, PRD88 (2013) 111501*

*Beuf, PRD85, (2012) 034039*

# ***SUMMARY***

***CGC is a systematic approach to high energy collisions***

***it has been used to fit a wealth of data; ep, eA, pp, pA, AA***

***Leading Log CGC works (too) well for a qualitative/semi-quantitative description of data, NLO is needed***

***Need to eliminate/minimize late time/hadronization effects***

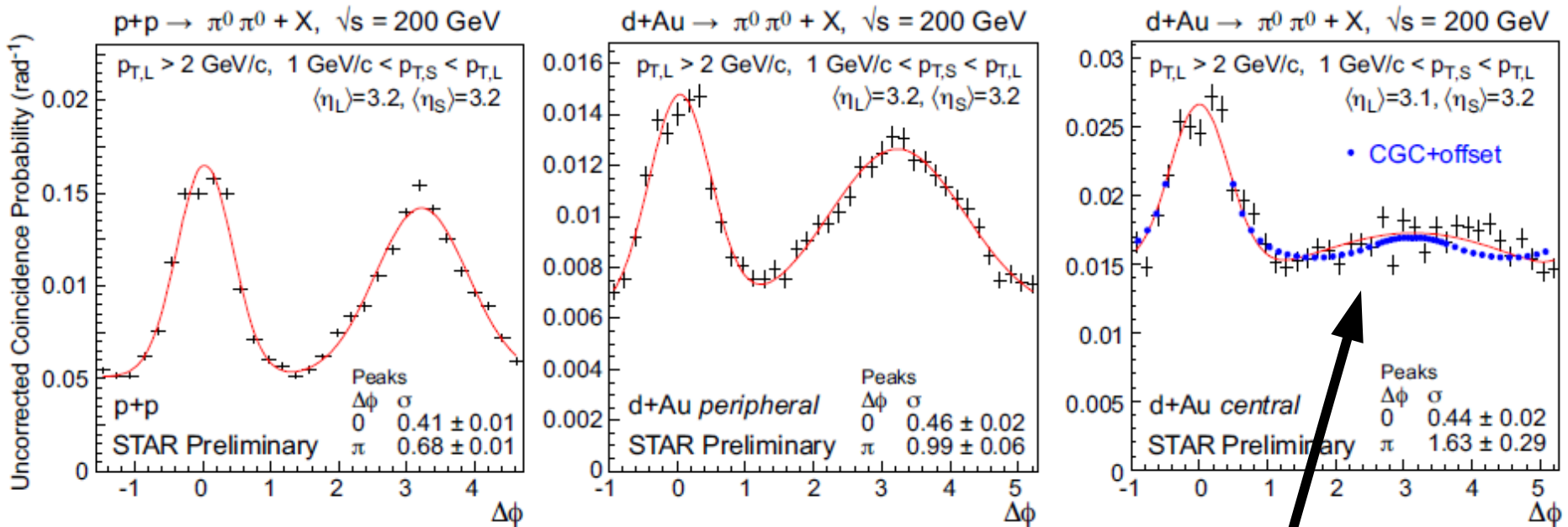
***Azimuthal angular correlations offer a unique probe of CGC***

***3-hadron/jet correlations should be even more discriminatory***

***an EIC is needed for precision CGC studies***

# *di-hadron correlations are a sensitive probe of CGC*

Recent STAR measurement (arXiv:1008.3989v1):



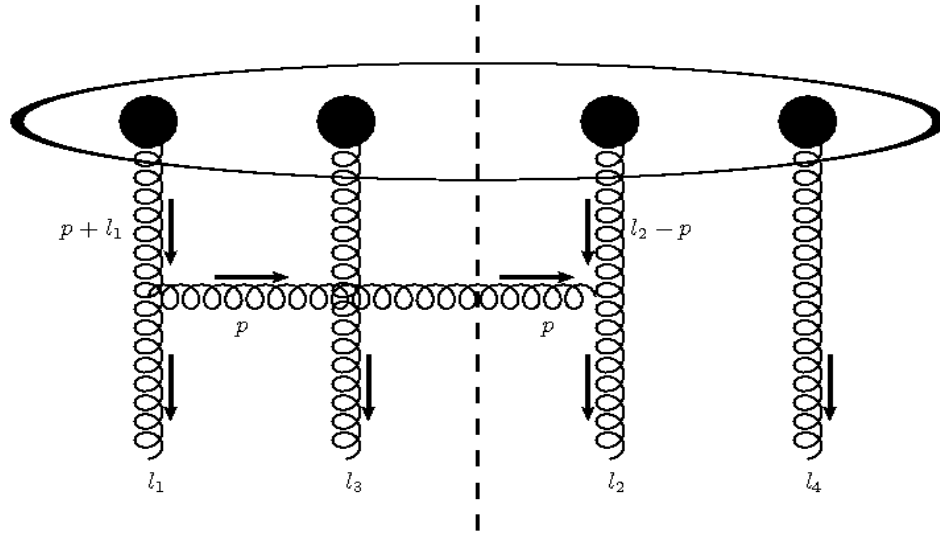
**saturation effects  
de-correlate  
the hadrons**

Marquet, NPA (2007), Albacete + Marquet, PRL (2010)  
 Tuchin, NPA846 (2010)  
 A. Stasto + B-W. Xiao + F. Yuan, PLB716 (2012)  
 T. Lappi + H. Mantysaari, NPA908 (2013)

shadowing+energy loss: Z. Kang, I. Vitev, H. Xing, PRD85 (2012) 054024

# JIMWLK evolution of quadrupole: linear regime

## BJKP equation



$\mathcal{O}(\Lambda^4)$  : 4-gluon exchange

*J. Jalilian-Marian, PRD85 (2012) 014037*

$$\begin{aligned} \frac{d}{dy} \hat{T}_4(l_1, l_2, l_3, l_4) &= \frac{N_c \alpha_s}{\pi^2} \int d^2 p_t \left[ \frac{p^i}{p_t^2} - \frac{(p^i - l_1^i)}{(p_t + l_1)^2} \right] \cdot \left[ \frac{p^i}{p_t^2} - \frac{(p^i - l_2^i)}{(p_t + l_2)^2} \right] \\ &\quad \hat{T}_4(p_t + l_1, l_2 - p_t, l_3, l_4) + \dots \\ &- \frac{N_c \alpha_s}{(2\pi)^2} \int d^2 p_t \left[ \frac{l_1^2}{p_t^2 (l_1 - p_t)^2} + \{l_1 \rightarrow l_2, l_3, l_4\} \right] \hat{T}_4(l_1, l_2, l_3, l_4) \end{aligned}$$

**this will de-correlate the produced partons at high  $p_t > Q_s$**

# **di-hadron production in DIS**

$$\gamma^*(\mathbf{k}) \mathbf{p} \rightarrow \mathbf{q}(\mathbf{p}) \bar{\mathbf{q}}(\mathbf{q}) \mathbf{X}$$

$$\mathcal{A}^\mu(k, q, p) = \frac{i}{2} \int \frac{d^2 l_\perp}{(2\pi)^2} d^2 x_\perp d^2 y_\perp e^{i(p_\perp + q_\perp - k_\perp - l_\perp) \cdot y_\perp} \\ e^{i l_\perp \cdot x_\perp} \bar{u}(q) \Gamma^\mu(k^\pm, k_\perp, q^-, p^-, q_\perp - l_\perp) v(p) \\ [\mathbf{V}(x_\perp) \mathbf{V}^\dagger(y_\perp) - 1]$$

with

**quadrupoles**

$$\Gamma^\mu \equiv$$

$$\frac{\gamma^-(\not{q} - \not{l} + m) \gamma^\mu (\not{q} - \not{k} - \not{l} + m) \gamma^-}{p^- [(q_\perp - l_\perp)^2 + m^2 - 2q^- k^+] + q^- [(q_\perp - k_\perp - l_\perp)^2 + m^2]}$$

*F. Gelis and J. Jalilian-Marian, PRD67 (2003) 074019*

*Zheng + Aschenauer + Lee + Xiao, PRD89 (2014)7, 074037*

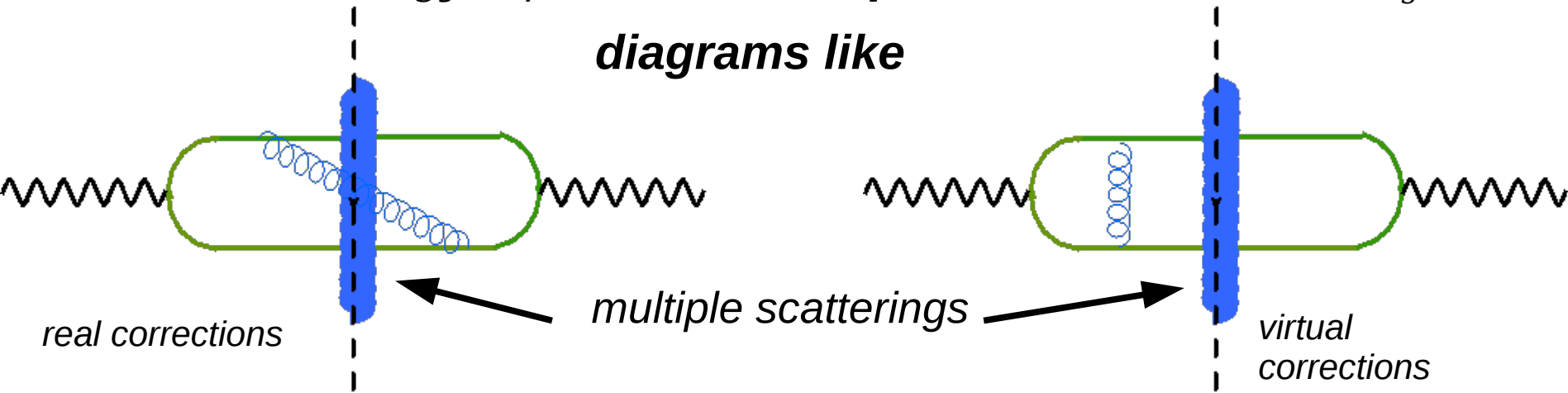
# DIS total cross section: energy (x) dependence

*recall the parton model was scale invariant, scaling violation (dependence on  $Q^2$ ) came after quantum corrections -  $O(\alpha_s)$*

*what we have done so far is to include high gluon density effects but no energy dependence yet*

*to include the energy dependence, need quantum corrections -  $O(\alpha_s)$*

**diagrams like**



**x dependence of dipole cross section:  
BK/JIMWLK evolution equation**

***NLO corrections  
recently computed***

***Extensive phenomenology at HERA***